ABSTRACT

Singular Poisson-Kähler geometry of stratified Kähler spaces

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A *stratified Kähler space* is a stratified symplectic space together with a complex analytic structure which is compatible with the stratified symplectic structure; in particular each stratum is a Kähler manifold in an obvious fashion. The notion of stratified Kähler space establishes an intimate relationship between nilpotent orbits, singular reduction, invariant theory, reductive dual pairs, Jordan triple systems, symmetric domains, and pre-homogeneous spaces. The purpose of the talk is to illustrate the significance of stratified Kähler spaces.

Examples of stratified Kähler spaces abound. The closure of a holomorphic nilpotent orbit carries a normal Kähler structure. Symplectic reduction carries a Kähler manifold to a normal stratified Kähler space in such a way that the sheaf of germs of polarized functions coincides with the ordinary sheaf of germs of holomorphic functions. Projectivization of holomorphic nilpotent orbits yields exotic stratified Kähler structures on complex projective spaces and on certain complex projective varieties including complex projective quadrics. Other examples come from certain moduli spaces of holomorphic vector bundles on a Riemann surface and variants thereof; in physics language, these are spaces of conformal blocks. Still other physical examples are reduced spaces arising from angular momentum.

In the world of singular Poisson-Kähler geometry, reduction after quantization coincides with quantization after reduction: For a stratified symplectic space, the concept of stratified polarization, which is defined in terms of an appropriate Lie-Rinehart algebra, encapsulates polarizations on the strata and, moreover, the behaviour of the polarizations across the strata. Exploiting the notion of stratified Kähler space, one can prove that, given a Kähler manifold, reduction after quantization coincides with quantization after reduction in the sense that not only the reduced and unreduced quantum phase spaces correspond but the invariant unreduced and reduced quantum observables as well.